Linhas de transmissão - Parte 4a

Example: # chapter 06 Transmission line

Considere uma L.T. com as seguintes características

$V_{sg} = 100\angle 0°$
$Z_g = R_g = 50\Omega$
$f = 100\,MHz$

Linha sem perdas com: $L = 0{,}25\,\mu H/m$
$C = 100\,pF/m$
$\ell = 10\,m$

Cargas: $Z_L = R_L = 0;\ 25;\ 50;\ 100;\ \infty\ \Omega$

Determine:
a) Coeficiente de reflexão na carga $\Gamma_L = (Z_L - Z_0)/(Z_L - Z_0)$
b) Relação de onda estacionária $s = (1+|\Gamma_L|)/(1-|\Gamma_L|)$
c) Impedância de entrada $Z_{in}(z) = Z_0\left[Z_L + jZ_0\tan(\beta(\ell - z))\right]/\left[Z_0 + jZ_L\tan()\right]$
d) gráficos de tensão e corrente ao longo da linha

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{0{,}25\times10^{-6}}{100\times10^{-12}}} = 50\,\Omega$$ imped. característica

$$\beta = \omega\sqrt{LC} = 2\pi\cdot100\times10^{6}\sqrt{(0{,}25\times10^{-6})(100\times10^{-12})} = 3.141592\ rd/m$$

$\boxed{\beta = \pi}$ constante de propagação

Comprimento de onda $\lambda = \frac{2\pi}{\beta}$

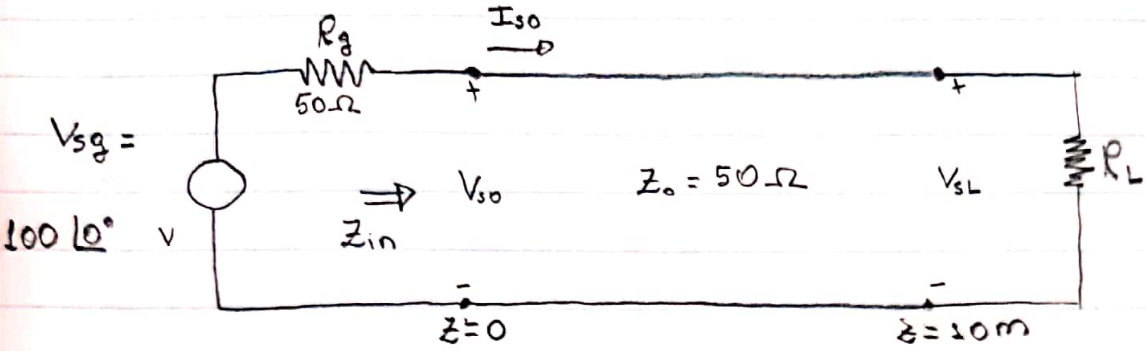
$\boxed{\lambda = 2m}$

Comprimento da linha em unidades de λ:

$\boxed{\ell = 5\lambda}$

fase da onda no final da linha

$$\boxed{\beta \ell = 10\pi}$$



Impedância de entrada LT sem perdas ⇒ eq. 16

$$Z_{in}(z) = Z_0 \cdot \frac{Z_L + jZ_0 \tan[\beta(\ell - z)]}{Z_0 + jZ_L \tan[\beta(\ell - z)]}$$

como entrada ⇒ $z = 0$

$$Z_{in}(z) = 50 \cdot \frac{R_L + j50 \cancel{\tan(10\pi)}^{\,0}}{50 + jR_L \cancel{\tan(10\pi)}_{\,0}}$$

$$Z_{in}(z) = R_L \quad \Rightarrow \quad \boxed{Z_{in} = R_L} \; *$$

Coeficiente de reflexão na carga ⇒ $z = \ell$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{R_L - 50}{R_L + 50}$$

Relação de onda estacionária: s

de 20:

$$s = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$$

Scanned with CamScanner

tensões e correntes ao longo da linha

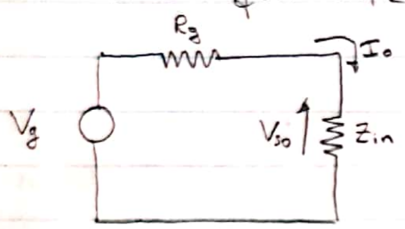
de (17) e (18):

$$|V_s(z)| = |V_{s0}^+| \cdot |1 + \Gamma(z)| = |V_{s0}^+| \cdot |1 + \Gamma_L e^{j2\beta(z-\ell)}|$$

$$|I_s(z)| = |I_{s0}^+| \cdot |1 - \Gamma(z)| = |I_{s0}^+| \cdot |1 - \Gamma_L e^{j2\beta(z-\ell)}|$$

$$|I_s(z)| = \frac{|V_{s0}^+|}{Z_0} |1 - \Gamma_L e^{j2\beta(z-\ell)}|$$

Circuito equivalente p/ encontrar V_{s0}^+ :



mas $Z_{in} = Z_L = R_L$

$$V_g = I_0 R_g + V_{s0}$$

$$I_0 = \frac{V_{s0}}{R_L} \Rightarrow V_g = V_{s0} \frac{R_g}{R_L} + V_{s0}$$

$$V_g = V_{s0} \left(\frac{R_g + R_L}{R_L} \right) \qquad V_{s0} = \frac{R_L}{R_g + R_L} \cdot V_{sg}$$

$$V_{s0} = 100 \cdot \frac{R_L}{50 + R_L}$$

mas $V_{s0} = V_s(0)$

Logo

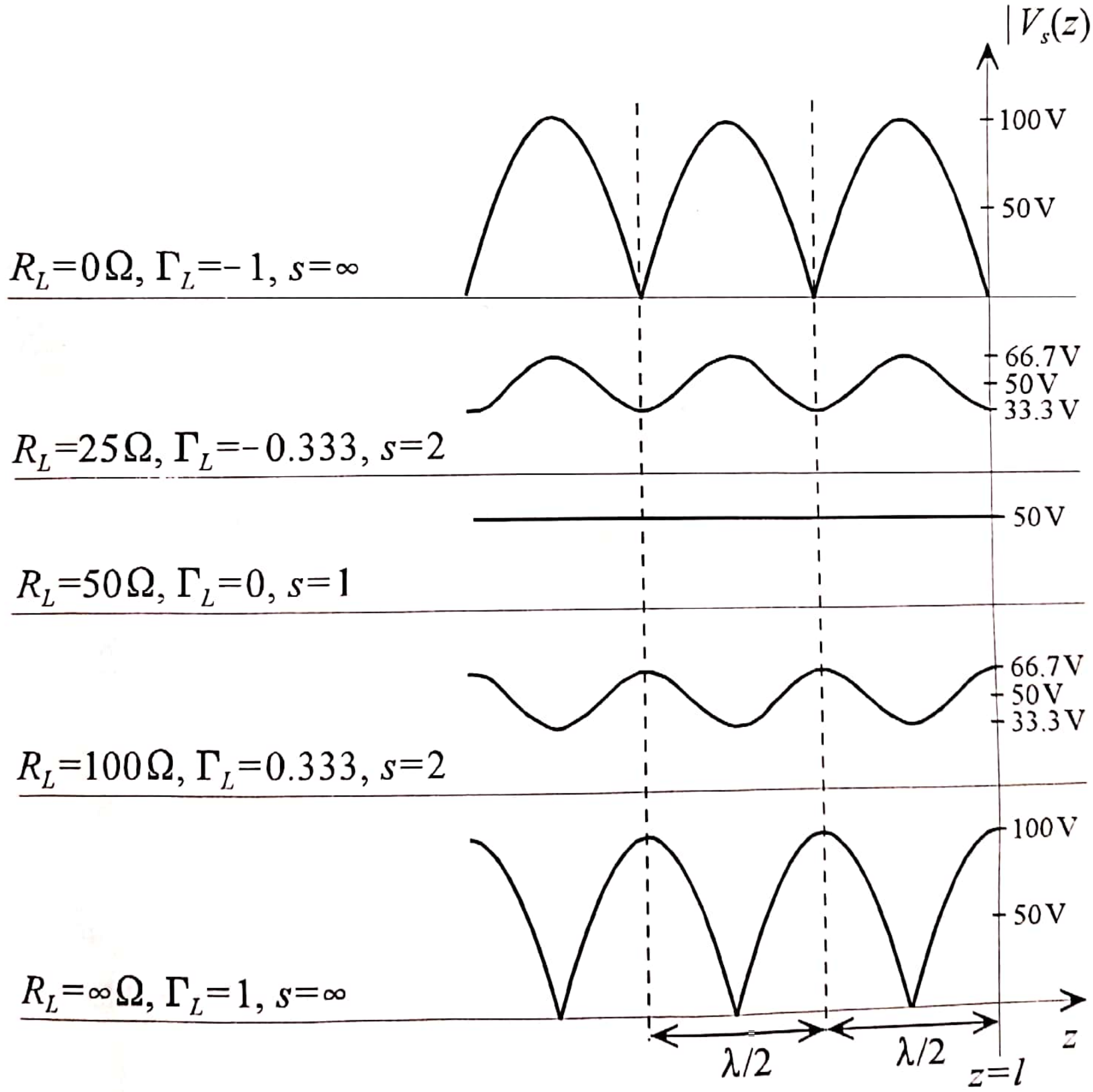
$$\frac{R_L}{R_g + R_L} \cdot V_{sg} = |V_{s0}^+| \cdot |1 + \Gamma_L \underbrace{e^{-j2\beta\ell}}_{=1}|$$

$$|V_{s0}^+| = \frac{100}{|1 + \Gamma_L|} \cdot \left(\frac{R_L}{R_g + R_L} \right)$$

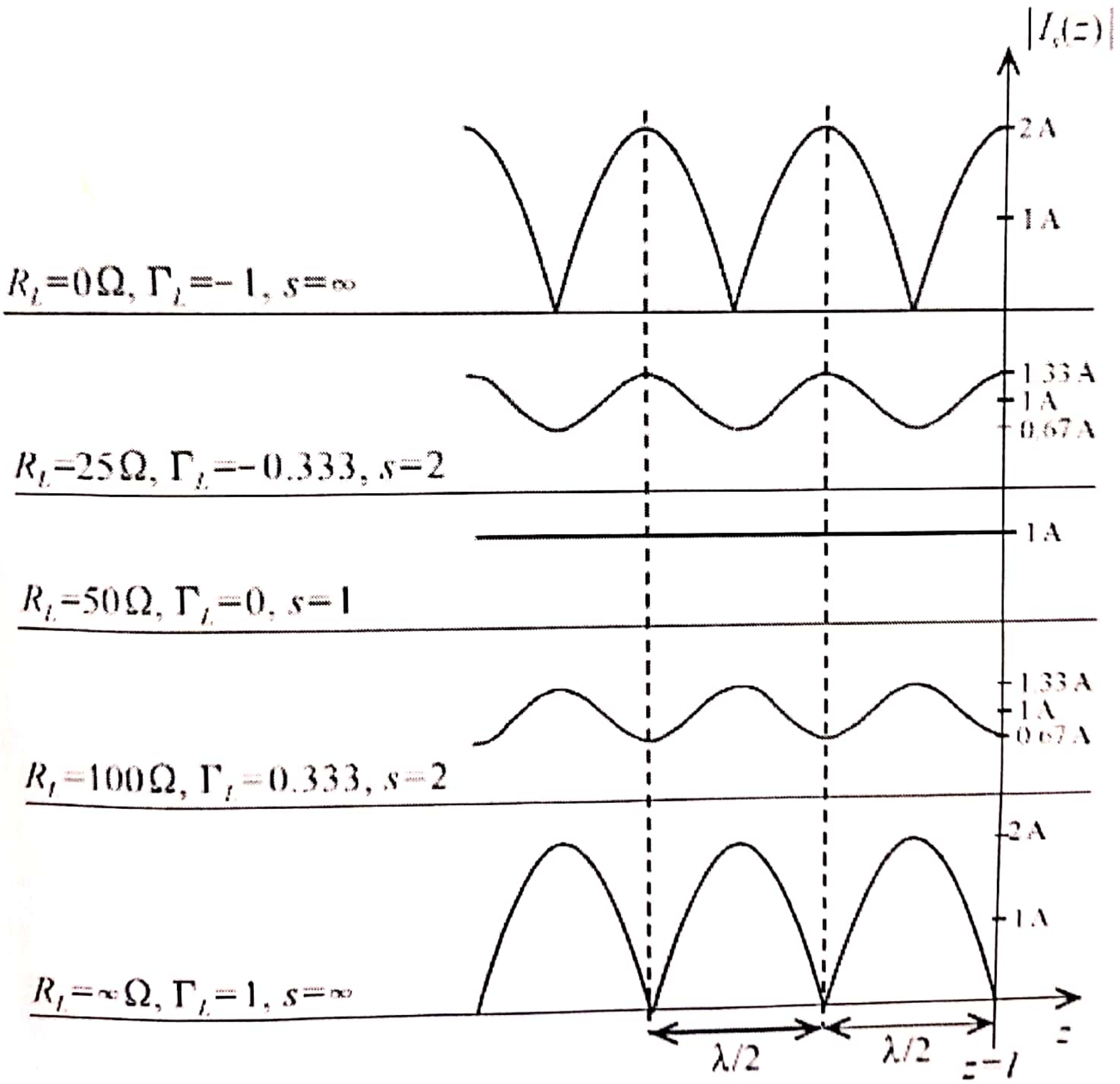
como

$$\Gamma_L = \frac{R_L - 50}{R_L + 50} \Rightarrow |V_{s0}^+| = \frac{100}{\left(1 + \frac{R_L - 50}{R_L + 50}\right)} \cdot \left(\frac{R_L}{R_L + 50}\right) = \frac{100 R_L}{2 R_L} = 50$$

R_L (Ω)	$\lvert V_s(z)\rvert_{max}$ (V)	$\lvert V_s(z)\rvert_{min}$ (V)	$\lvert V_s(l)\rvert$ (V)
0	100	0	0
25	66.7	33.3	33.3
50	50	50	50
100	66.7	33.3	66.7
∞	100	0	100



R_L (Ω)	$\lvert I_s(z)\rvert_{max}$ (A)	$\lvert I_s(z)\rvert_{min}$ (A)	$\lvert I_s(l)\rvert$ (A)
0	2	0	2
25	1.33	0.67	1.33
50	1	1	1
100	1.33	0.67	0.67
∞	2	0	0



Assim :

$$|V_s(z)| = |V_{s0}^+| \, |1 + \Gamma(z)|$$

$$|V_s(z)| = 50 \left| 1 + \Gamma_L e^{j2\beta(z-\ell)} \right|$$

$$|V_s(z)|_{max} = 50 \left(1 + |\Gamma_L| \right)$$

$$|V_s(z)|_{min} = 50 \left(1 - |\Gamma_L| \right)$$

$$|I_s(z)| = \frac{|V_{s0}^+|}{Z_0} \cdot \left| 1 - \Gamma_L e^{j2\beta(z-\ell)} \right|$$

$$= \frac{50}{50} \left| 1 - \Gamma_L e^{j2\beta(z-\ell)} \right|$$

$$|I_s(z)| = \left| 1 - \Gamma e^{j2\beta(z-\ell)} \right|$$

$$|I_s(z)|_{max} = 1 + |\Gamma_L|$$

$$|I_s(z)|_{min} = 1 - |\Gamma_L|$$

$$|I_s(\ell)| = |1 - \Gamma_L|$$